

Analytical Solutions for river-aquifer systems

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Who uses analytical solutions?

- We all do!
- If you use a K, its probably inferred from analytical solutions to Darcy's equation
- The most sophisticated and detailed models are built on 1d radial solutions
- How do we use them?

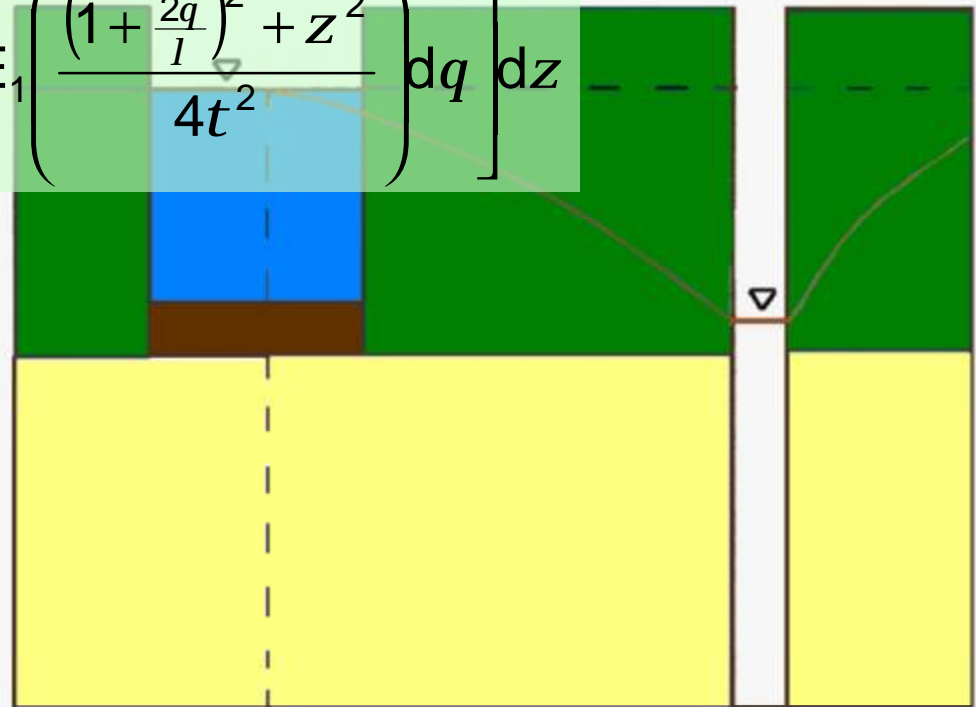


It's all about the conceptual model



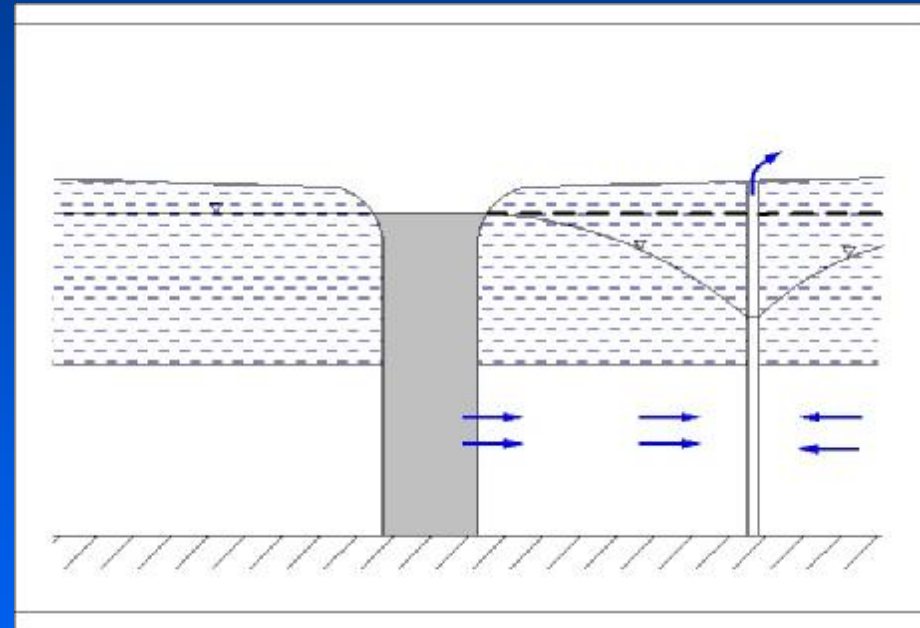
- How do we turn this?

$$\frac{q}{Q} = \frac{I}{4p} \int_{\frac{x_1}{a}}^{\frac{x_2}{a}} \left[E_1 \left(\frac{1+z^2}{4t^2} \right) - \int_0^\infty e^{-q} E_1 \left(\frac{\left(1 + \frac{2q}{I}\right)^2 + z^2}{4t^2} \right) dq \right] dz$$

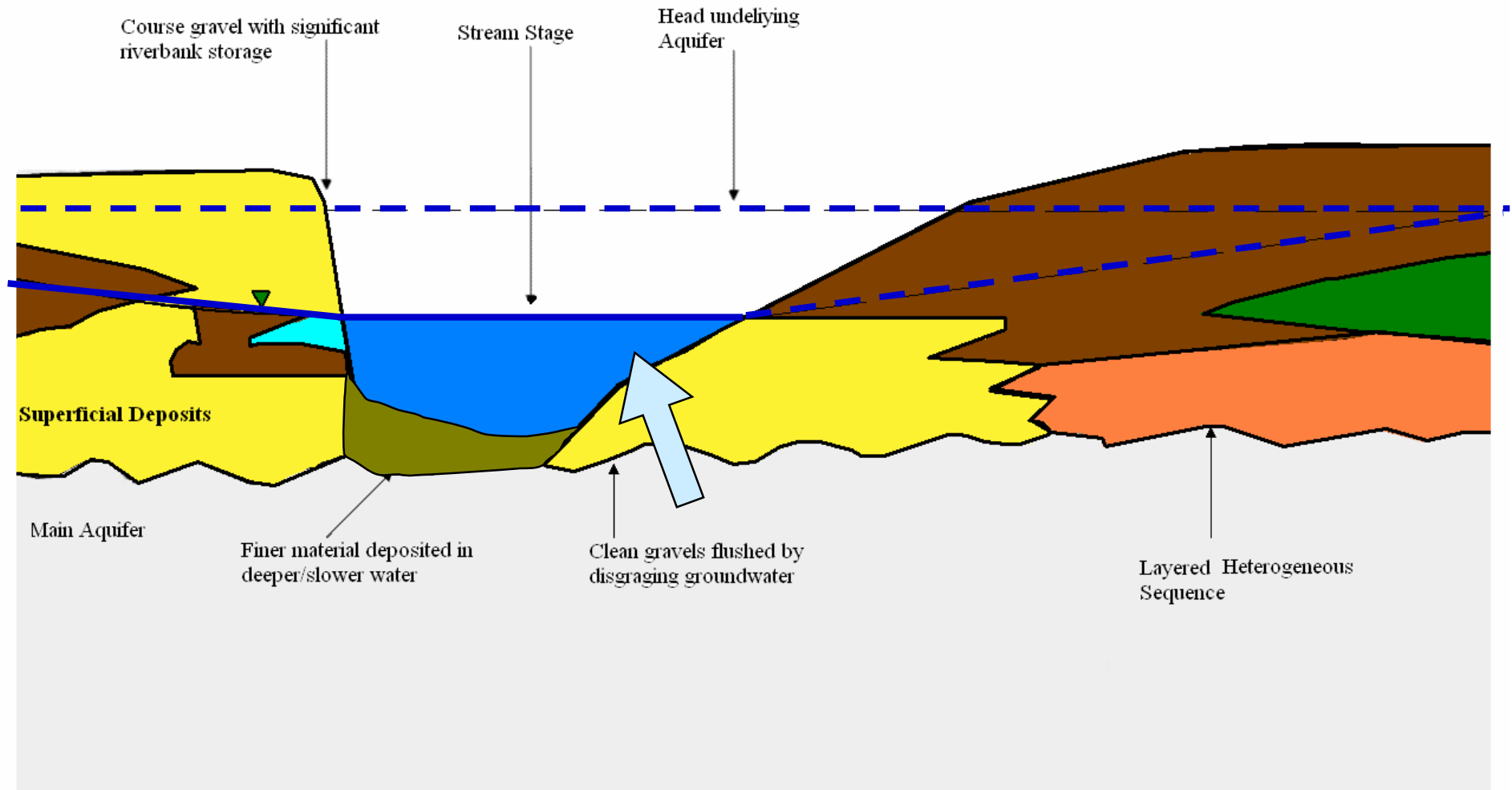


Pumping test assumptions

- The aquifer is confined
- The aquifer has a seemingly infinite areal extent
- The aquifer is homogeneous isotropic and uniform thickness
- The rest water level is horizontal over the area that will be influenced by the test
- The discharge rate is constant
- The well is fully penetrating and flow is horizontal
- The diameter of the well is small and there is no storage in the well
- The water removed from storage is discharged simultaneously with decline in head



What are real streams like?





Problems!

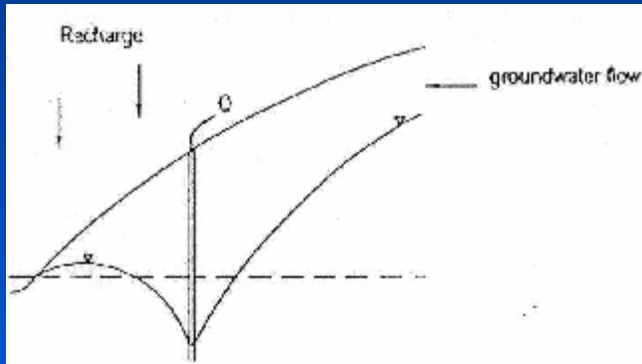
- No recharge or groundwater flow!
- What's on the other side of the river?
- What about sediments?
- My river isn't straight!
- I'm interested in a sensitive reach
- I've got two rivers!
- It's a leaky multi-layer aquifer!



These are common problems!

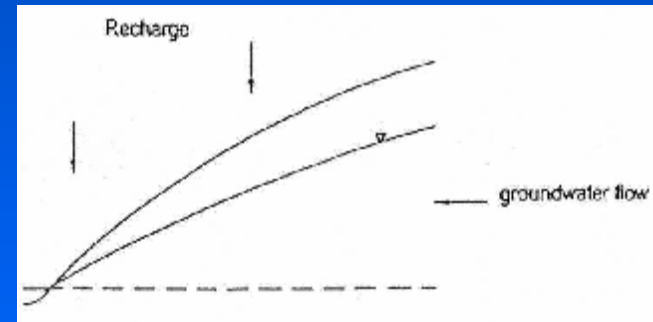
- We still use analytical solutions – nothing new
- Need to understand how we address approximations to the conceptual model

Superposition

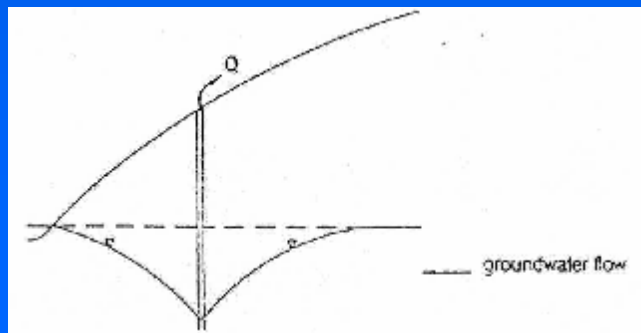


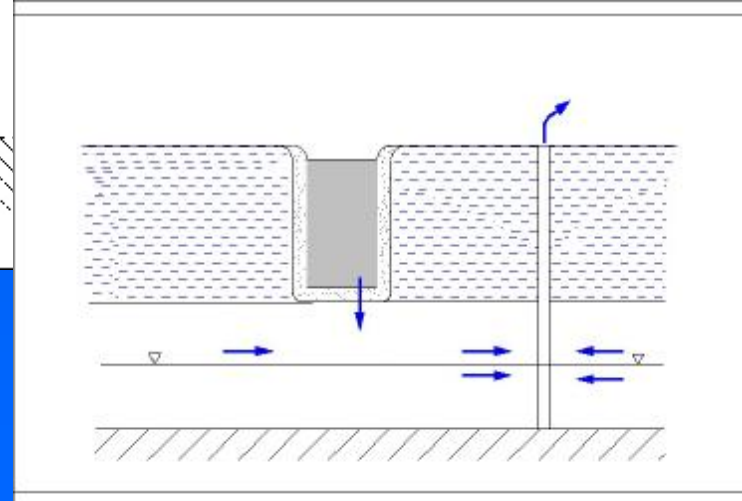
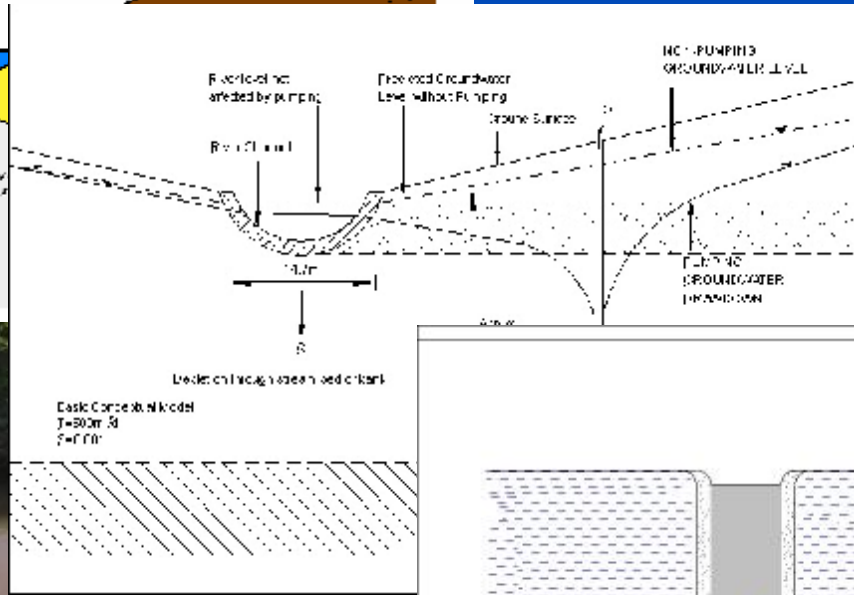
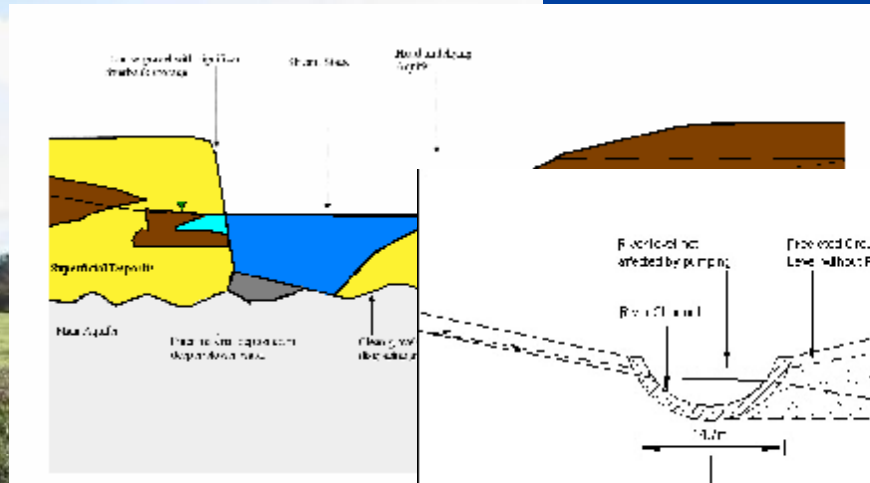
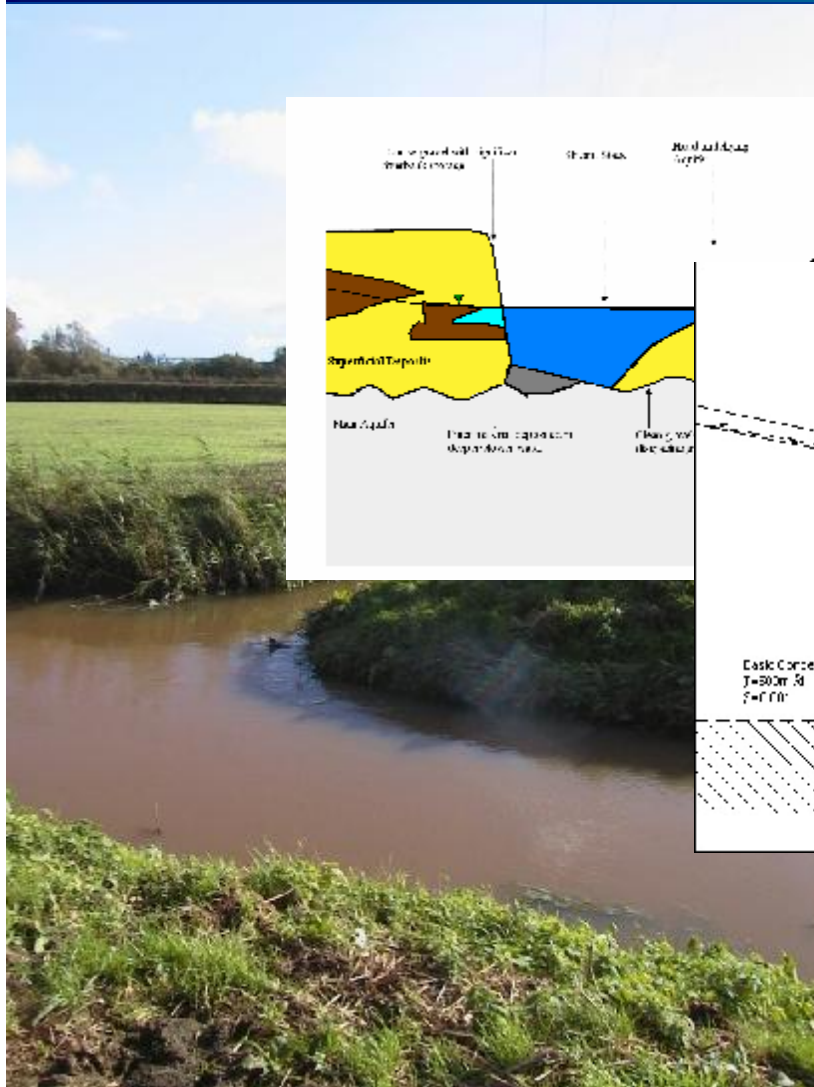
Physical water table, or superposition of heads (relative to static water level) due to recharge and base flow.

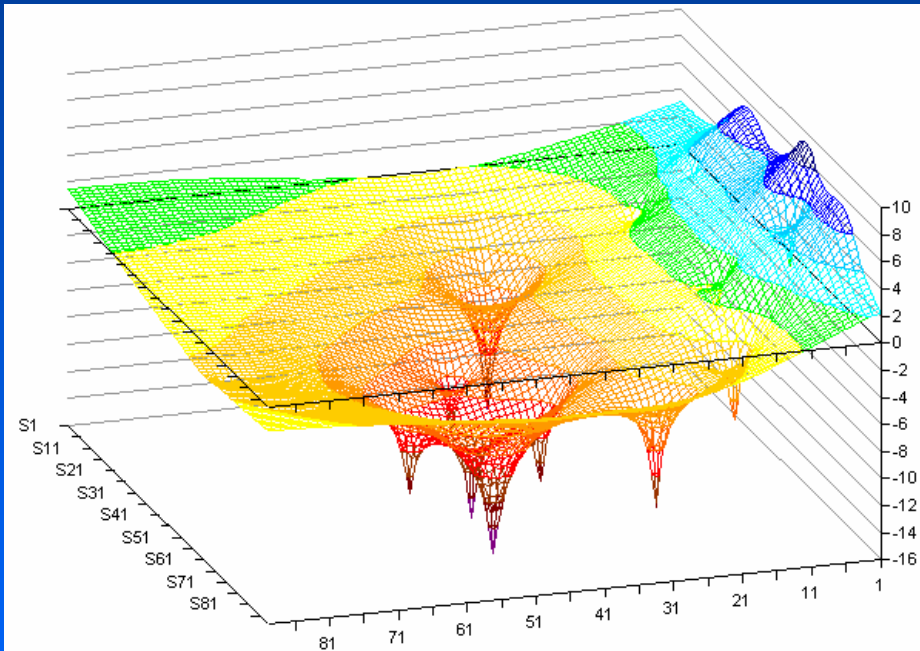
Water table prior to abstraction with head above static water table due to recharge and base flow.



Drawdown due to pumping, relative to static water level.



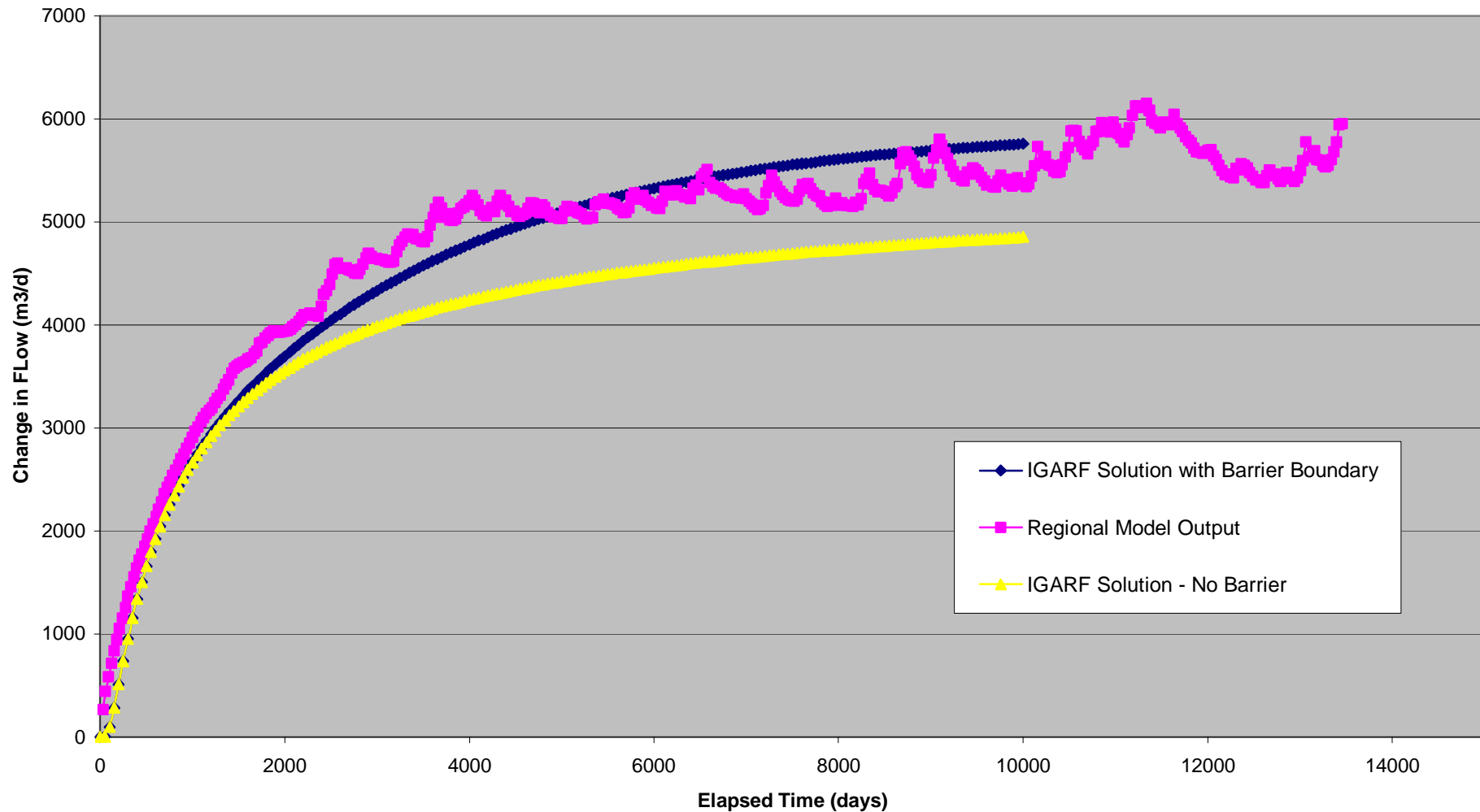




- Good on spatial variability
- Length scales are still likely to be large (>100m)
- Complex to setup and process output
- Good for detailed catchment scale planning



Comparison of analytical and numerical approaches





When are analytical solutions useful

- If the system is uncertain, analytical solutions are very good at quick and detailed evaluation of simple systems
 - Geometry - simple
 - Properties - homogeneous
 - Processes – linear
- If the system is well characterised and the results need to reflect detail, analytical solutions will oversimplify



What cannot be done

- **Non-linear effects**
 - Disconnected reaches
 - Shallow unconfined
 - Drawdown at wells
- **Leaky multilayered systems**
 - too much geometry
- **Heterogeneous rivers/aquifers**
 - too many properties!
- **Water quality**
 - too much process (transport)

- Analytical solutions and scoping tools will often give as good an answer as more sophisticated and expensive tools
- They need to be matched to the conceptual model
- They are a tool for a modeller – not a replacement!
- Choose the most appropriate tools for the job!



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